# **Transitivity: A Comment on Block and Barnett**

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**Abstract:** In a response to Block and Barnett (2012), this paper clarifies some misunderstandings about the concept of transitivity and shows its relation to rationality, asynchronicity of choice, and indifference analysis. It is demonstrated that Block and Barnett, contrary to their self-interpretation, do not in fact attack transitivity but the assumption of stable preferences. It is argued that the stability of preferences assumption cannot be easily dismissed.

Key words: transitivity, rationality, consistency, indifference analysis, stable preferences

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In my comment to Block and Barnett (2012) I attempt to clarify the meaning of the concept of transitivity, which is portrayed imprecisely in their paper; I will show that their attack is aimed at the assumption of stable preferences, rather than transitivity. My paper proceeds as follows. Section 1 clarifies misunderstandings about transitivity. Section 2 focuses on how transitivity is related to rationality and shows that disagreement between the Austrians and mainstream economists is only apparent. The issue of how the atemporal concept of transitivity translates into the fact that choices are asynchronic is dealt with in Section 3 - it is shown that the assumption of the stability of preferences is crucial in this context. The stability of preferences is further discussed in Section 4, while Section 5 briefly examines the implications of transitivity for the indifference curves. Section 6 concludes the paper.

#### **1** What is transitivity?

Transitivity (far from being an "equation" as Block and Barnett (2012, 238) claim) is a property applicable to binary relations. Some relations satisfy this property and some do not. Consider, for instance, the following two relations: 'to be a brother of' and 'to be a biological father of'. For any Tom, Dick, and Harry, if 'Tom is a brother of Dick' and 'Dick is a brother of Harry', then 'Tom is a brother of Harry'; the relation 'to be a brother of' is thus transitive. The relation 'to be a biological father of' is not transitive, since if 'Tom is a biological father of Dick' and 'Dick is a biological father of Harry', the implication 'Tom is a biological father of Harry' does not hold.

Block and Barnett wrongly treat transitivity as a property of *characteristics of objects* (or "elements", such as velocity, height, etc.) rather than *relations*, and fail to distinguish among various different types of relations – in particular, between "to be greater than" and "to be preferred to" (they use the same symbol ">" for both). I assume that they are ready to admit that the relation "to be greater than" satisfies transitivity; however, I would be surprised if they found the following proposition unobjectionable: 'a given economic actor, Jones, prefers driving 70 miles per hour to 60 miles per hour, and 60 miles per hour to 50 miles per hour. This implies that he also prefers 70 miles per hour to 50 miles per hour.' But this is, in fact, one of the possible interpretations of their claim (Block a Barnett 2012, 238).

The problem examined by Block and Barnett may be restated as follows: is it correct to assume that the relation "to be preferred to" satisfies transitivity? First note that (contrary to what Block and Barnett suggest) the relation "to be preferred to" as used in mainstream economics is atemporal – it does not refer to a specific moment or time period. Block and Barnett do not seem to have a problem with transitivity applied to this atemporal preference relation (Block a Barnett 2012, 242). After all, transitivity is widely used in Austrian economics (although somewhat secretly): whenever the concept of a 'preference scale' is mentioned, the transitivity of preference is implicitly adopted.<sup>2</sup> The differences, according to the authors, between the Austrians and mainstream economists emerge when (i) transitivity is identified with rationality, and when (ii) time is introduced into the picture. I first discuss the relationship between rationality and transitivity.

#### 2 The meaning of rationality

According to Block and Barnett, the transitivity of preferences is for the mainstream economist "necessary, at least if rationality is to be preserved; for the praxeologist, in sharp contrast, it constitutes only one of several options, all of which may be characterized as 'rational'" (Block a Barnett 2012, 238). I claim that this difference between the praxeologist and the mainstream economist is only apparent, as it is based on a misunderstanding. The key lies in different *definitions* of rationality: for Block and Barnett rationality seems (they do not introduce an explicit definition) to mean 'reasonability'; in the dictionary of a mainstream economist, however, rationality is defined in terms of completeness and the transitivity of preferences, and may be different from 'reasonability' (Mas-Colell, Whinston, a Green 1995, 6). In other words, it is conceivable for a mainstream economist, just as for Block and Barnett, to consider the idea of preferences violating transitivity 'reasonable'. Therefore, as long as a praxeologist and mainstream economist adopt the same language, they are in agreement on this point.

In the light of the preceding paragraph it becomes clear that the so called 'money pump' argument is not intended to be a "knockout blow against the Austrian contention that there is nothing irrational about non-transitivity" (Block a Barnett 2012, 240). The money pump, considered by some mainstream economists "not [...to be a] necessarily convincing argument for rationality" which could be "easily criticized" (Rubinstein 2006, 26), should rather

 $<sup>^{2}</sup>$  The intuition is clear: if the transitivity of preference is violated, then preferences are cyclical and alternatives cannot be ordered on a scale.

illustrate the Alchian-Friedman evolutionary argument that people with intransitive preferences might have a hard time to survive in market competition (Gintis 2009). Nothing in mainstream economics is said about the 'reasonability' or otherwise of a person being deprived of money.

#### **3** Transitivity and time

Having discussed the relationship between transitivity and rationality, I now attempt to clarify the mainstream position with respect to asynchronicity of choice. As acknowledged by both the Austrians and mainstream economists, an observer does not have direct access to the preferences of an agent; all he can see are the agent's choices. Can an observer tell whether an agent's preferences satisfy transitivity only by observing the agent's choices? As proved by Arrow (1959), he can – if certain assumptions are met:

(i) *Revealed preference*: if *x* is chosen while *y* is also feasible, then *x* is preferred to *y*.(ii) *Stability of preferences*: preferences do not change over time.

Under these two assumptions, an agent has transitive preferences if and only if his choices are consistent in the following sense: if x is chosen while y is feasible, then when y is chosen x must no longer be feasible.

Now, the assumption of revealed preference is non-controversial, as it is adopted also by the Austrians under the label 'demonstrated preference'.<sup>3</sup> What seems to be problematic is the assumption of the stability of preferences: if it were not for this assumption, it would be impossible to distinguish between intransitivity and a preference change (Rosenberg 1992). Block and Barnett are thus right by pointing out that the identification of consistency of choice with transitivity requires that the stability of preferences is *assumed* as a non-testable proposition. Under this assumption, mainstream economists interpret all observed inconsistency as a preference change, then they have to accept transitivity as a non-testable proposition, rather than fighting it! Their inability to identify transitivity as their ally rather than their enemy is due to the fact that they do not distinguish between 'transitivity of

<sup>&</sup>lt;sup>3</sup> Rothbard (1977) maintains that 'revealed preference' of the mainstream is "seemingly similar but actually quite different concept" from 'demonstrated preference' of the Austrians; careful reading of his argument reveals that it is not the case and that in fact he only rejects the assumption of stable preferences.

preferences' on the one hand and 'consistency of choice' on the other. In consequence, they are forced to introduce the temporal aspect into the concept of transitivity, which blurs who their real enemy is – namely, the stability of preferences. In the next section I will attempt to show that there actually exist strong arguments for adopting this assumption, which Block and Barnett fail to address.

#### 4 Stability of preferences

An important defence of the concept of stable preferences is provided by Becker and his coauthors (Michael a Becker 1973; Stigler a Becker 1977). Consider the standard demand function: x = (p, I, T), where x is a quantity of a good, p is a price vector, I is income, and T are preferences or "tastes". Assume that we observe a change in x; whenever we are unable to explain the change in x by the changes in p and I, we have to content ourselves with the "explanation" that preferences have changed. Becker et al. noticed that we had no theory of how preferences form or change, hence the inverted commas around the term explanation. The research strategy implied by the approach of Becker et al. is thus to try to identify parameters in the 'residual category' T, expose them to the light, and make them explicit codeterminants of behaviour. Only when explanation by the parameters that influence behaviour in a predictable manner fails do we turn to the "explanation" that preferences have changed.

Where Block and Barnett seem to be satisfied with the explanation "Green changes his mind about his trip to Canada" (Block a Barnett 2012, 241), a mainstream economist who follows Becker's research program, far from denying that unexplainable changes in mind can occur, would look for a hidden parameter that might have played a role in the change of Green's behaviour (is it perhaps additional information that made Green change his mind?).

There is also another – although closely related – argument for sticking to the assumption of stable preferences, which is best illustrated by an example: assume that market demand for a trip to Canada falls. It is rather unlikely that this fall is due to the fact that many Greens simply changed their minds. Therefore, if we admit that economics is mainly concerned with market-level phenomena such as this, we may accept the assumption of stable preferences as a plausible assumption. Is it not the case that also Austrians explain changes in market level phenomena in terms of a change of some parameter rather than as a result of unexplainable but, for some reason, correlated changes in people's preferences?

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#### **5** Indifference curves

Finally, I will briefly discuss the relationship between transitivity and indifference curves. Block and Barnett reproduce the standard proof that the transitivity of the indifference relation implies that indifference curves do not cross. The crossing of indifference curves is nothing but a graphical representation of intransitivity: the two are just different means of representing one idea. Therefore, it is simply not true that "indifference curves are compatible only with transitivity" (Block a Barnett 2012, 239) and that "[i]f transitivity fails, according to this logic, then so must indifference curves" (Block a Barnett 2012, 246). Whether preferences are transitive or not has no bearing whatsoever on the *existence* of indifference curves; it translates only into their *properties*.

#### 6 Conclusion

Transitivity may indeed be an "economic travesty" as Block and Barnett (2012, 247) claim; but they fail to demonstrate that this is the case. The assumption that they really object to is not transitivity but the stability of preferences. Here, I have attempted to clarify the meaning of transitivity and to show that the assumption of stable preferences, attacked by Block and Barnett, cannot be dismissed so easily.

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